Class XI- MATHEMATICS

Chapter-3 : TRIGONOMETRIC FUNCTIONS

Hand out of Module 2/3

In this module we are going to learn about

- **Sign of trigonometric functions**
- **Domain and range of trigonometric functions**
- **Behaviour of trigonometric functions in different quadrants.**
- **Graph of trigonometric functions**

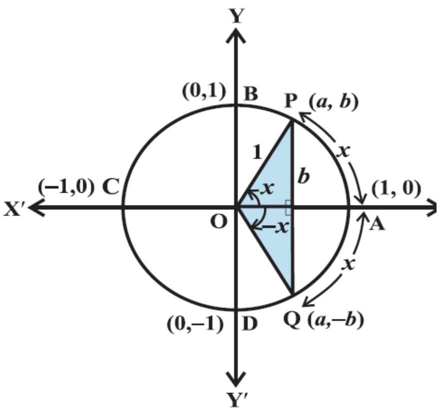
Sign of trigonometric functions

Let P(a, b) be a point on the unit circle with centre at the origin such that $\angle$ AOP = x.

We define cos x = a and sin x = b.

If, $\angle$AOQ = – x, then the coordinates of point Q will be (a, –b).

Therefore **cos (– x) = cos x** and **sin (– x) = – sin x**



Since for every point P (a, b) on the unit circle,

$-1 \le a \le 1$ and $-1 \le b \le 1$.

That is $-1 \le \cos x \le 1$ and $-1 \le \sin x \le 1$ for all x.

We know that a is positive in first and fourth quadrant.

Similarly, b is positive in first and second quadrant.

Therefore,

sin x is positive for $0 < x < \pi$ and negative for $\pi < x < 2\pi$.

Similarly, cos x is positive for $0 < x < \frac{\pi}{2}$, negative for $\frac{\pi}{2} < x < \frac{3\pi}{2}$ and positive for $\frac{3\pi}{2} < x < 2\pi$.

Sign of trigonometric functions in different quadrants.

	I	II	III	IV
sin x	+	+	–	–
cos x	+	–	–	+
tan x	+	–	+	–
cosec x	+	+	–	–
sec x	+	–	–	+
cot x	+	–	+	–

Domain and range of trigonometric functions

From the definition of sine and cosine functions, we observe that they are defined for all real numbers. Further, we observe that for each real number *x*, $-1 \le \cos x \le 1$ and $-1 \le \sin x \le 1$

Function	Domain	Range
sin x	R	[–1, 1]
cos x	R	[–1, 1]
tan x	R –{x:x = (2n+1) $\frac{\pi}{2}$, n ∈ Z}	R
cosec x	R– { x: x = nπ , n ∈ Z}	R – (– 1, 1)
sec x	R –{x: x = (2n+1) $\frac{\pi}{2}$, n ∈ Z}	R – (– 1, 1)
cot x	R– {x: x = nπ , n ∈ Z}	R

Behaviour of trigonometric functions in different quadrants.

	I quadrant	II quadrant	III quadrant	IV quadrant
sin x	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to –1	increases from –1 to 0
cos x	decreases from 1 to 0	decreases from o to -1	increases from –1 to 0	increases from 0 to 1
tan x	increases from 0 to ∞	increases from - ∞ to 0	increases from 0 to ∞	increases from -∞ to 0
cosec x	decreases from ∞ to 1	increases from 1 to ∞	increases from $-\infty$ to -1	decreases from $-1\ to -\infty$
sec x	increases from 1 to ∞	increases from $-\infty$ to -1	decreases from -1 to $-\infty$	decreases from ∞ to 1
cot x	decreases from ∞ to 0	decreases from $0\ to -\infty$	decreases from ∞ to 0	decreases from $0\ to -\infty$

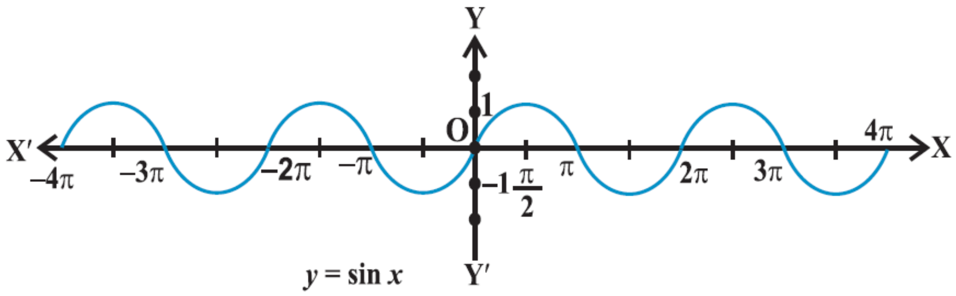
Graph of trigonometric functions

The values of sin *x* and cos *x* repeat after an interval of 2π.

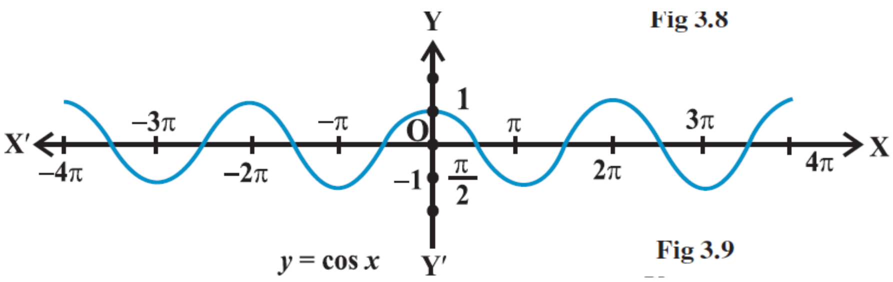
Hence, values of cosec *x* and sec *x* will also repeat after an interval of 2π.

The values of tan *x* and cot x repeat after an interval of π.

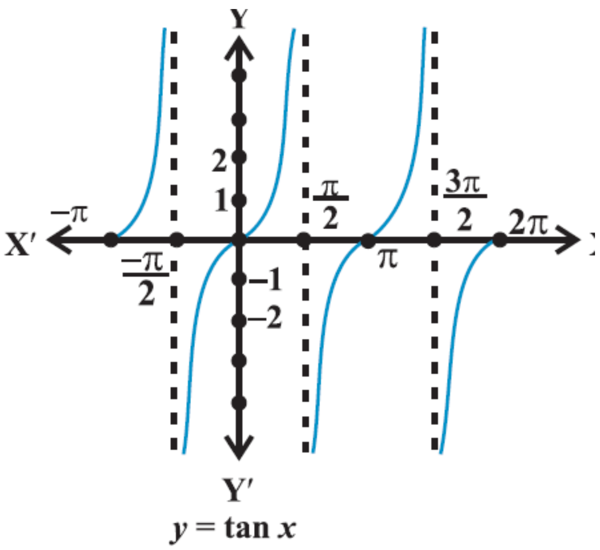
1) y= sin x



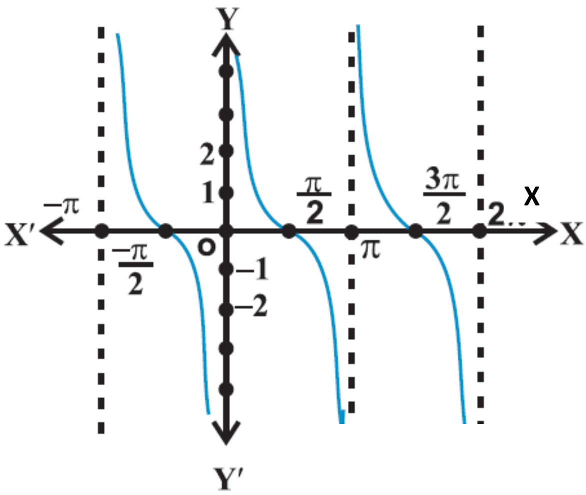
2) y= cos x



3).**y = tan x**

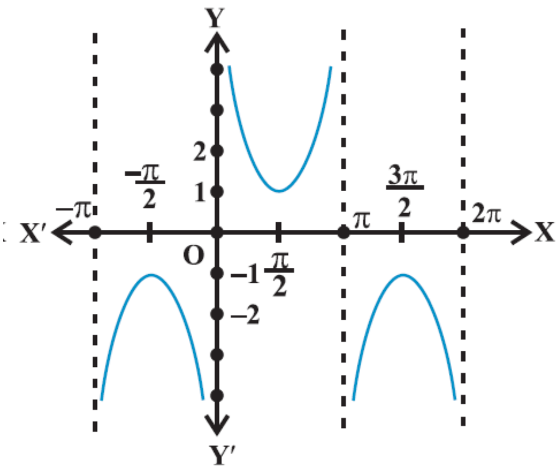


4).**y = cot x**



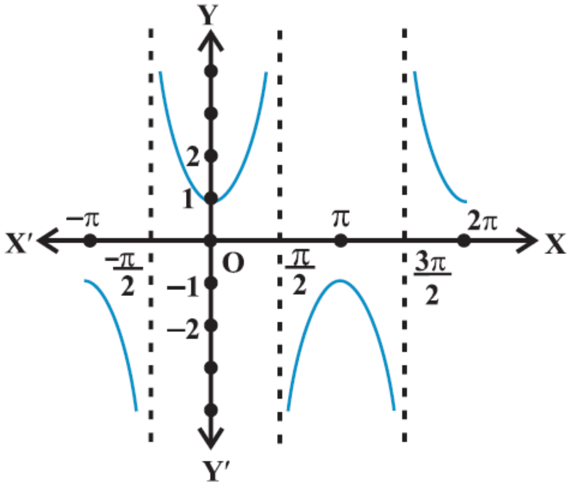
$y = \cot x$

5) y = cosec x



$y = \text{cosec}\, x$

6) y = sec x

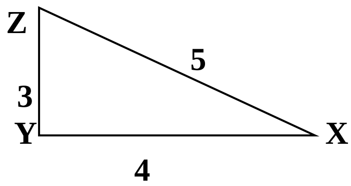


$y = \sec x$

Example 1:

Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, x lies in second quadrant

Solution: $\sin x = \frac{3}{5}$, therefore $\operatorname{cosec} x = \frac{5}{3}$



In figure, YZ = 3 units, XZ = 5 units.

Also, by using Pythagoras theorem, XY = 4 units

Since x lies in second quadrant, cos x, sec x, tan x and cot x will be negative.

Therefore, $\cos x = \frac{-4}{5}$, $\sec x = \frac{-5}{4}$, $\tan x = \frac{-3}{4}$ and $\cot x = \frac{-4}{3}$

Example 2 :

Find the value of cos (–1710°).

Solution: We know that values of cos x repeats after an interval of 2π or 360°.

Therefore, cos (–1710°) = cos (–1710° + 5 × 360°) = cos (–1710° + 1800°) = cos 90° = 0

Example 3:

Find the value of sin $\frac{-31\pi}{3}$

Solution: We know that sin(-x) = - sinx

Also, values of sin x repeat after an interval of 2π.

Therefore, sin $\frac{-31\pi}{3}$ = - sin $\frac{31\pi}{3}$

$$= -\sin\left(10\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}.$$

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